**1) Decision Variables :**

The key decision variables are:

* *Xij*: a binary variable equal to 1 if truck iii takes route jjj, and 0 otherwise.
* tij: the time required for truck iii to travel route jjj, which can vary depending on peak hours (if considered).
* di​: the total duration of the trip for truck iii.
* k: total number of available trucks.
* TMAX​: the maximum time taken by any truck to complete its deliveries and return to the base.

**2) Objective Function: Minimize Total Time**

The objective is to **minimize the total time** required for all trucks to complete their deliveries and return to the base. Therefore, we are looking to minimize the return time of the **last truck** (the one that takes the longest).

The objective function becomes:

Minimize TMAX

Where TMAX is the maximum time taken by any truck to return to the base, which can be written as:

TMAX = max (d1, d2, ……dk)

​

Here, **di** represents the total time taken by truck **i**to complete its entire route.

**3) Constraints (based only on time)**

**a) Assigning Trucks to Routes**

Each delivery point must be assigned to exactly **one truck**. This ensures that every point is covered, but also allows trucks to optimize their routes by visiting nearby points. This can be formulated as:

Une image contenant Police, horloge, Graphique, typographie

Description générée automatiquement

Th This means that each delivery point j is handled by one and only one truck i

***b) Optimizing Routes by Visiting Nearby Points***

***trucks can deliver to nearby points before returning to the base. This optimization happens at the algorithmic level. Once a truck is assigned a primary point B, the algorithm will look for nearby points C, D, etc., that are close enough to be visited before returning to the base.***

***In practice, this optimization step doesn't require changes to the mathematical constraints but rather an enhancement to the algorithm that generates the routes. It ensures that trucks take the most efficient route based on proximity.***

***The algorithm can be summarized as:***

* ***If truck i is at point B, check for nearby points C,D,… before returning to the base.***
* ***Minimize the total time di by including these nearby points if it reduces the overall return time.***

**c) Travel Time with Traffic Considerations**

The time to travel between points may vary depending on traffic conditions. We can model this as:

Une image contenant texte, Police, capture d’écran, ligne

Description générée automatiquement​

This accounts for the possibility that travel times between certain points may change based on the time of day. The total duration di​ of each truck's trip will take this into account.

**d) Returning to the Base**

Every truck must return to its starting point (the base) after completing its deliveries. This is a standard constraint in route optimization:

Une image contenant texte, Police, Graphique, capture d’écran

Description générée automatiquement

This constraint ensures that every truck returns to the base after completing its assigned deliveries.

The **objective function** remains minimizing TmaxT\_{max}Tmax​, and the **constraints** ensure that every point is covered by a truck and that all trucks return to their starting point.